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CURRENT RECTIFICATION OF QUANTUM NANO DIODES IN SUPERCONDUCTING STATE

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ABSTRACT

Nano materials can only be described by quantum laws. In this work quantum treatment for two nano metal contacts in a superconducting state is done. A useful expression for the input and output current is found by using the notion of the current density in quantum mechanics. The conditions required by this contact to act as a rectifier and amplifier are discussed .It is found that certain restrictions should be imposed on the potential and the wave number of the metal contact to act as a rectifier.

Keywords: *Quantum, Superconducting, Rectifier, Current density, Transition temperature.*

I. INTRODUCTION

The phenomenon of superconductivity, in which the electrical resistance of certain materials completely vanishes at low temperatures, is one of the most interesting phenomenon in condensed matter physics [1]. In 1911 Kamerlingh Onnes and one of his assistants discovered the phenomenon of superconductivity while studying the resistance of metals at low temperatures [2]. Superconductivity refers to a complex of phenomena which occurs in a wide variety of metals and alloys, below a transition temperature () which currently ranges up to ~ 150K (50% of room temperature) [3]. Of the various constituent phenomena, the two which are conceptually most important are macroscopic diamagnetism and persistent super currents. Superconductivity is a state of matter below a certain critical temperature [4]. Heavily doped semiconductor can become superconducting such as Ge or Si. Nor the noble metals (Cu,Ag,Au) neither the alkalis (Li, Na, K, Rb, Cs, Fr) present a superconducting phase transition at least above a few mille Kelvin [5]. In general, good conductors are not good superconductors (meaning that they do not have high critical temperatures). The number of conducting electrons in a metal is of the order of per cm^3. In a semiconductor at room temperature these are of the order of and in a heavily doped semiconductor this number (in the same units) is around [6]. Super conductor current diodes and transistors are commonly used in electronic circuits in order to extract the original signal from the carrier wave so as to be displayed on the display unit [7]. The diode act in these circuits as a rectifier for it allows current to flow in a certain direction and prevent it to flow in the opposite direction [8]. Section (2) is devoted for theoretical back ground. The contribution is in section (3) and (4). In this contribution quantum mechanics is used to describe the behavior of diodes. Since nano materials can only be described by quantum laws,. Discussion and conclusion are exhibited in sections (5) and (6).

II. THEORETICAL BACK GROUND (Junction Diode)

Any diode is formed from p-type and n-type joined together to form one crystal. In this case free electrons diffuse from the n- type, due to the diffusion process to fill holes in the in p-type. Due to this process positive ions are formed in the n-type near the junction, while negative ions are accumulated in the p-type. As a result potential barrier is formed to prevent the diffusion of more electrons from the n- type to the p-type. If one connects a battery such that its positive pole is connected to the p- type, while its negative pole is attached to the n- type. The negative pole force free electrons to move towards the p- type and as a result electric current flows through the junction. This current is called forward current and its intensity is given by:

(1)

Where is called the saturation current, (T) is the absolute temperature and (V) is the applied voltage with V =|V|. The diode can also be connected in such a way that the negative pole of the battery is attached to the p- type and the positive Pole is connected to the n- type.

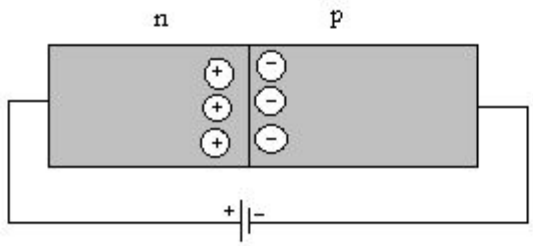

Figure (1): Reverse Bias Connection

In this case the current flows since the poles of the battery attract charges away from the p- n junction this increases the accumulation of ions and the potential barrier which prevents current from flowing. This connection is termed reverse bias connection and the current flows is given by

(2)

Where the applied voltage

$$V = -|V| \tag{3}$$

Causes only the saturation current is which is negligibly small to flow. Hence the current flows are practically equal to zero.

III. THE QUANTUM NANO DIODE

The diode is usually made from n-type and p-type semi conductor, when these are fused together a potential hill is formed. In this work a potential hill can alternatively be formed by bringing two metals of different work functions ϕ_1 , ϕ_2 in direct contact. The height of the potential hill (or step) be comes

$$V_o = \phi_2 - \phi_1 \tag{4}$$

By appropriately choosing the origin of the x-axis to be at the junction of the two contacts, and choosing the origin of the potential axis to be at( )level, Schrödinger equation for the two metal contacts become:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \tag{5}$$

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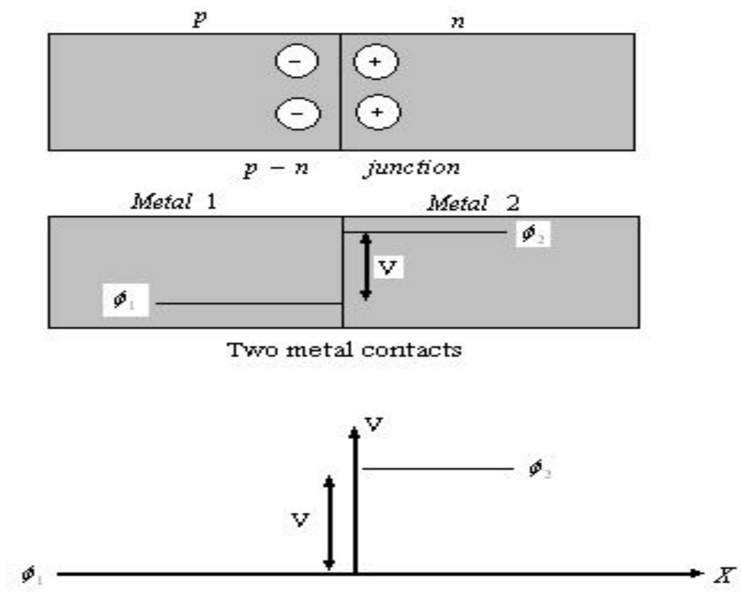

Figure (2): the potential step of p-n junction and two metal contacts

But since the potential is described by the function

$$\begin{aligned} V(x) &= 0 \qquad for \qquad x < 0 \\ V(x) &= V_o \qquad for \qquad x > 0 \\ V(x) &= \phi_2 - \phi_1 \end{aligned} \tag{6}$$

Therefore Schrödinger equation for metal one, where become

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \tag{7}$$

Where $$k^2 = \frac{2m}{\hbar^2}E \tag{8}$$

For metal two, where Schrödinger equation takes the form

$$\frac{d^2\psi}{dx^2} + k_1^{\,2}\psi = 0 \tag{9}$$

Where $$k^2{}_1 = \left[\frac{2m}{\hbar^2}(E - V_o)\right] \tag{10}$$

The general solutions are

$$\begin{aligned} \psi &= Ae^{ikx} + Be^{-ikx} \qquad for \qquad x < 0 \\ \psi &= Ce^{ik_1x} + De^{-ik_1x} \qquad for \qquad x > 0 \end{aligned} \tag{11}$$

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If are multiplies the wave functions by the time dependent factor $\exp(-iEt/\hbar)$, one can interpret the terms Ae^{ikx} *and* Ce^{ik_1x} as waves prorogating in the +x direction, and the terms Be^{-ikx} *and* De^{-ik_1x} as waves prorogating in the –x direction. Since the particles is incident from the left on the barrier at x= 0, there cannot be a wave prorogating in the –x direction, in region $x > 0$, and hence one must put D=0. Therefore the solution becomes

$$\begin{aligned} \psi_1 = \psi = Ae^{ikx} + Be^{-ikx} \quad & for \quad x < 0 \\ \psi_2 = \psi = Ce^{ik_1x} \quad & for \quad x > 0 \end{aligned} \tag{12}$$

Where the terms

$$\psi_i = Ae^{ikx}, \quad \psi_r = Be^{-ikx}, \quad \psi_t = Ce^{ik_1x} \tag{13}$$

Represents incident, reflected and transmitted wave respectively. The corresponding current densities can be obtained by using the expression [9, 10].

$$J_\psi = nev = evn = ev|\psi|^2$$

$$J_\psi = ev\psi^*\psi = e\psi^* v\psi = e\psi^* \frac{p}{m}\psi$$

$$J_\psi = e\psi^* \frac{\hat{p}}{m}\psi = e\psi^*\left(\frac{\hbar}{im}\nabla\psi\right)$$

$$J_\psi = e\psi^*\left(\frac{\hbar}{im}\frac{d\psi}{dx}\right) \tag{14}$$

Where one takes the real part Re

$$J = \mathrm{Re}\, J_\psi = \mathrm{Re}\left[e\psi^* \frac{\hbar}{im}\frac{d\psi}{dx}\right] \tag{15}$$

But before determining J, it is important to determine the value of the unknown parameters A, B and C. since ψ is continuous at x=0, therefore

$$\begin{aligned} \psi_1(0) &= \psi_2(0) \\ A + B &= C \end{aligned} \tag{16}$$

Applying the continuity of the derivatives at the same point one gets.

$$\left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}$$

$$\left[ikAe^{ikx} - ikBe^{-ikx}\right]_{x=0} = \left[ik_1 Ce^{ik_1 x}\right]_{x=0}$$
$$ik(A - B) = ik_1 C$$
$$A - B = \frac{k_1}{k} C \tag{17}$$

Adding equations (16) and (17) yields

$$2A = \left(1 + \frac{k_1}{k}\right) C$$
$$C = \frac{2k}{k + k_1} A \tag{18}$$

Using (18) in (15) one gets

$$B = C - A$$
$$B = \frac{k - k_1}{k + k_1} A \tag{19}$$

The incident current density can be obtained from (14) and (15) in the from

$$J_i = \mathrm{Re}\left[\frac{e\hbar}{im} \psi_i^* \frac{\partial \psi_i}{\partial x}\right]$$
$$= \mathrm{Re}\left[e \frac{\hbar}{im} A^* e^{-ikx} A \frac{\partial e^{ikx}}{\partial x}\right]$$
$$J_i = \frac{e\hbar k}{m} A^* A = \frac{e\hbar k}{m} |A|^2 \tag{20}$$

Similarly the reflected current density becomes

$$J_r = \mathrm{Re}\left[\frac{e\hbar}{im} \psi_r^* \frac{\partial \psi_r}{\partial x}\right] = \mathrm{Re}\left[\frac{e\hbar}{im} B^* e^{ikx} B \frac{\partial}{\partial x} e^{-ikx}\right]$$
$$J_r = \mathrm{Re}\left[-\frac{e\hbar k}{m} B^* B\right] = \frac{e\hbar k}{m} |B|^2 \tag{21}$$

Also the transmitted current density can be obtained using the same procedures to get

$$J_t = \mathrm{Re}\left[\frac{e\hbar}{im} \psi_t{}^* \frac{\partial \psi_t}{\partial x}\right]$$
$$J_t = \frac{e\hbar k_1}{m} |C|^2 \tag{22}$$

If one need the two metal contacts to act as a rectifier then the transmitted current density should become very small compared to or vanish, while the incident current density becomes non zero. This can be achieved by considering being zero in equation (10), i.e.

$$k_1 = \left[\frac{2m}{\hbar^2}(E - V_o)\right]^{\frac{1}{2}} = 0 \tag{23}$$

In this case

$$E = V_o \tag{24}$$

Hence vanishes according to equation (22). The transmitted current can also vanish if (note equation (22))

$$k = 0 \qquad and \qquad C = 0 \tag{25}$$

But also vanishes as shown by equation (20) which is physically not acceptable.

The rectification can also take place when

$$J_i >> J_t \tag{26}$$

This means the diode must prevent current flow from left to right. In view of equations (20) and (22) one gets

$$\frac{e\hbar k|A|^2}{m} >> \frac{e\hbar k_1|C|^2}{m} \tag{27}$$

But from equation (18)

$$|C|^2 = \frac{4k^2}{(k + k_1)^2}|A|^2 \tag{28}$$

Therefore

$$\begin{aligned}
&k|A|^2 >> \frac{4k_1k^2}{(k + k_1)^2}|A|^2 \\
&1 >> \frac{4k_1k}{(k + k_1)^2} \\
&(k + k_1)^2 >> 4k_1k \\
&k^2 + 2kk_1 + k_1^2 >> 4kk_1 \\
&k^2 + k_1^2 >> 2kk_1 \\
&k^2 + k_1^2 - 2kk_1 >> 0 \\
&(k_1 - k)^2 >> 0
\end{aligned} \tag{29}$$

This means that either

$$
\begin{aligned}
&(k_1 - k) >> 0 \\
&-(k_1 - k) >> 0 \\
&k - k_1 >> 0
\end{aligned}
\tag{30}
$$

Hence either

$$
\begin{aligned}
&k_1 >> k \qquad I.e \qquad k_1^2 >> k^2 \\
&k >> k_1 \qquad I.e \qquad k^2 >> k_1^2
\end{aligned}
\tag{31}
$$

With the aid of equations (8) and (10) one obtains

$$
\begin{aligned}
&\frac{2m}{\hbar^2}(E - V_o) >> \frac{2m}{\hbar^2}E \\
&-V_o >> 0 \\
&V_o << 0
\end{aligned}
\tag{32}
$$

Or

$$
\begin{aligned}
&\frac{2m}{\hbar^2}E >> \frac{2m}{\hbar^2}(E - V_o) \\
&E >> E - V_o \\
&0 >> -V_o \\
&V_o >> 0
\end{aligned}
\tag{33}
$$

But since from equation (6), then equation (32) is rejected while equation (33) is physically acceptable. This equation indicates that rectification can be obtained when the potential step

(See equation (6)) become extremely large, this means that the larger the difference between the works functions of the two metals the more the efficient the metal contact to rectify the alternating current, where the current is prevented to move from left to right. This condition can forms with figure (2) which shows that the electron must raise itself up the potential step to pass over the barrier.

If the current flows from right to left the incident, reflected and transmitted wave functions in regions I and II becomes

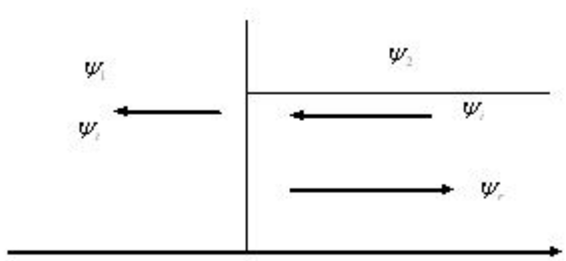

Figure (3): The Wave Functions in Regions I and II

$$\psi_i = De^{-ik_1x}, \psi_r = Ce^{ik_1x}, \psi_t = Be^{-ikx} \tag{34}$$

Hence

$$\begin{aligned} \psi_1 &= \psi_t = Be^{ikx} \\ \psi_2 &= \psi_r + \psi_i = Ce^{ik_1x} + De^{-ik_1x} \end{aligned} \tag{35}$$

Applying continuity of ψ *and* $\partial\psi/\partial x$ *at* $x = 0$ gives

$$\begin{aligned} &\psi_2(0) = \psi_1(0) \\ &C + D = B \\ &\left.\frac{\partial \psi_2}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} \\ &ik_1C - ik_1D = -kB \\ &-C + D = \frac{k}{k_1}B \end{aligned} \tag{36}$$

Addition of equations (36) yields

$$\begin{aligned} 2D &= \left(\frac{k + k_1}{k_1}\right)B \\ B &= \left(\frac{2k_1}{k + k_1}\right)D \end{aligned} \tag{37}$$

Similarly

$$C = B - D = \left(\frac{k_1 - k}{k_1 + k}\right)D \tag{38}$$

Utilizing equations (14) and (34), yields

$$\begin{aligned} J_i &= \frac{e\hbar k_1}{m}|D|^2 \\ J_r &= \frac{e\hbar k_1}{m}|C|^2 \\ J_t &= \frac{e\hbar k}{m}|B|^2 \end{aligned} \tag{39}$$

For rectification and amplification to take place the current flows from right to left should not diminish, this requires that the transmitted current should not be less than the incident current ie.

$$J_t \geq J_i \tag{40}$$

With the aid of equation (39) one gets

$$\begin{aligned} &\frac{e\hbar k}{m}|B|^2 \geq \frac{e\hbar k_1}{m}|D|^2 \\ &k|B|^2 \geq k_1|D|^2 \end{aligned} \tag{41}$$

Utilizing relation (37) yields

$$\begin{aligned} &k\left(\frac{2k_1}{k+k_1}\right)^2|D|^2 \geq k_1|D|^2 \\ &4kk_1^2 \geq k_1(k+k_1)^2 \\ &4kk_1 \geq k^2 + 2kk_1 + k_1^2 \\ &k_1^2 - 2kk_1 + k^2 \leq 0 \\ &(k_1 - k)^2 \leq 0 \\ &\pm(k_1 - k) \leq 0 \\ &k_1 - k \leq 0 \qquad k_1 \leq k \qquad k \geq k_1 \\ &k - k_1 \leq 0 \qquad k \leq k_1 \qquad k_1 \geq k \end{aligned} \tag{42}$$

Hence the requirements that

$$J_t \geq J_i \tag{43}$$

Can be satisfied by either

$$k \geq k_1 \qquad or \qquad k_1 \geq k \tag{44}$$

Or alternatively

$$k^2 \geq k_1^2 \qquad k_1^2 \geq k^2$$

In view of equations (8) and (10) one gets

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$$\frac{2m}{\hbar^2}E \geq \frac{2m}{\hbar^2}(E - V_o)$$

$$-V_o \leq 0 \qquad (45)$$

Or $$\frac{2m}{\hbar^2}(E - V_o) \geq \frac{2m}{\hbar^2}E$$

$$E - V_o \geq E \qquad -V_o \geq 0$$

$$V_o \leq 0 \qquad (46)$$

Equation (45) is physically acceptable since is assumed to be positive. While equation (46) is rejected since it is in conflict with the fact that is positive this result again conforms with figure (2), where electrons can sweep easily down the potential hill from right to left. Therefore rectification to take place the metal contact should prevent current to flow from left to right , i.e. , and this requires that as shown by equation(33) .At the same time the metal contact should allow current to flow from right to left, i.e. , which is satisfied if as shown by equation (45) . The two conditions are satisfied if

$$Vo >> 0 \qquad (47)$$

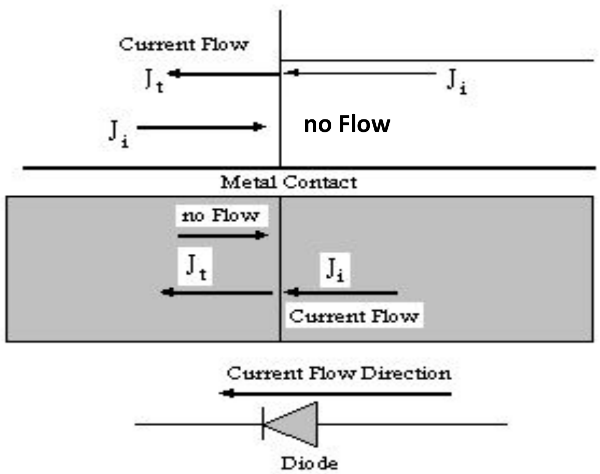

Figure (4): Direction of Current Flow

IV. CURRENT RECTIFICATION IN THE PRESENCE OF A MAGNETIC FIELD

If a magnetic field of a vector potential is applied the current density is given according to equation (22) and (14) by:

$$J = \frac{i\hbar e}{2m}\left[\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right] - \frac{-2e^2 A_m}{m}\psi^* \psi \qquad (48)$$

When the current flows from left to right the incident and transmitted current densities can be obtained with the aid of equation (13) to be:

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$$J_i = \frac{i\hbar e}{2m}\left[\left(A^* e^{-ikx}\right)\left(ikAe^{ikx}\right) - Ae^{ikx}\left(-ikA^* e^{-ikx}\right)\right] - \frac{2e^2 A_m |A|^2}{m}$$

$$J_i = -\frac{\hbar e}{m} k|A|^2 - \frac{2e^2 |A|^2 A_m}{m} \tag{49}$$

Since the current is positive one can take the modulus of this value i.e.

$$J_i = \frac{e}{m}|A|^2\left[\hbar k + 2eA_m\right] \tag{50}$$

Similarly the transmitted current takes the form

$$J_t = \frac{i\hbar e}{2m}\left[C^* e^{-ik_1 x}\left(ik_1 Ce^{ik_1 x}\right) - Ce^{ik_1 x}\left(-ik_1 C^* e^{-ik_1 x}\right)\right] - \frac{2e^2 A_m |C|^2}{m}$$

$$J_t = |J_t| = \frac{e}{m}|C|^2\left[\hbar k_1 + 2eA_m\right] \tag{51}$$

The system prevents current from flowing from left to right when

$$J_t << J_i \qquad I.e \qquad J_i >> J_t \tag{52}$$

According to (50) and (51)

$$\hbar k + 2eA_m >> \frac{4k^2}{\left(k + k_1^2\right)^2}\left[\hbar k_1 + 2eA_m\right] \tag{53}$$

If one assumes the particle energy E to be comparable to the potential step I.e. . In this case (see equation (10) Thus

$$\hbar k + 2eA_m >> 8eA_m$$

$$\hbar k >> 6eA_m \tag{54}$$

On the other hand when the current flows from right to left the incident and transmitted current densities can be obtained with the aid of equations (34) and (48) to get.

$$J_i = \frac{i\hbar e}{2m}\left[D^* e^{ik_1 x}\left(-ik_1\right)e^{-ik_1 x} - De^{-ik_1 x} D^*\left(ik_1\right)e^{ik_1 x}\right] - \frac{2e^2}{m} A_m |D|^2$$

$$J_i = \frac{e|D|^2}{m}\left[\hbar k_1 - 2eA_m\right] \tag{55}$$

Similarly

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$$J_t = \frac{i\hbar e}{2m}\left[B^* e^{ikx} B(-ik)e^{-ikx} - Be^{-ikx}(B^*)(ik)e^{ikx}\right] - \frac{2e^2 A_m}{m}|B|^2$$
$$J_t = \frac{e}{m}|B|^2\left[\hbar k - 2eA_m\right] \tag{56}$$

Using equation (38) yields

$$J_t = \frac{e}{m}\left(\frac{2k_1}{k+k_1}\right)^2 |D|^2\left[\hbar k - 2eA_m\right] \tag{57}$$

The system should allow the current flow from right to left and for existence rectification and amplification this requires that

$$J_t \geq J_i \tag{58}$$

Thus

$$\left(\frac{2k_1}{k+k_1}\right)(\hbar k - 2eA_m) \geq \hbar k_1 - 2eA_m$$
$$(2k_1)^2(\hbar k - 2eA_m) \geq (k+k_1)^2(\hbar k_1 - 2eA_m)$$
$$If \qquad k_1 \to 0 \tag{59}$$

As before

$$0 \geq k^2(-2eA_m)$$
$$2eA_m k^2 \geq 0 \tag{60}$$

The two conditions for rectification requires

$$\hbar k >> 6eA_m \qquad 2eA_m k^2 \geq 0 \tag{61}$$

These needs

$$k >> \frac{6eA_m}{\hbar}, \qquad A_m \geq 0 \tag{62}$$

V. DISCUSSION

Rectification can be achieved by two metal contacts in the absence and presence of applied magnetic field. In the absence of magnetic field rectification is possible if as shown by equation (33), while amplification is possible if as equation (45) shows. This is quite natural, since the electrons cannot flow from left to right due to the existence of potential barrier (see figure (4)). However the electrons can roll down the potential hill when they flow from right to left (see figure (2)). In the presence of a magnetic field equation (62) requires that rectification and amplification are possible when , . This is also obvious, since K is related to the electron energy as shown by equation (8). Applying large electric field the electron can gain enough energy to enable quantum nano diode to rectify and amplify current.

VI. CONCLUSIONS

The mathematical analysis done on the quantum expression for two metal contacts in a super conducting state shows the possibility of rectification and amplification if certain physical constraints are applied. In the absence of the magnetic field rectification and amplification is possible if the potential is positive and enormously large. In the presence of magnetic field the wave number should be related to the strength of the applied magnetic field.

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